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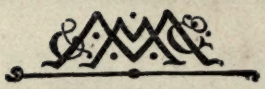
THE ADVANCED PART

OF A TREATISE ON THE

DYNAMICS OF A SYSTEM OF RIGID BODIES

BEING PART II. OF A TREATISE ON THE WHOLE SUBJECT.



THE ADVANCED PART

OF A TREATISE ON THE

DYNAMICS OF A SYSTEM OF RIGID BODIES.

BEING PART II. OF A TREATISE ON THE WHOLE SUBJECT.

With numerous Examples.

BY

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PREFACE.

THIS volume is intended to be a continuation of that already published as Part I. in 1882. The time occupied in its preparation has been longer than I had anticipated. This is partly due to the want of sufficient leisure, and partly also because as I proceeded with the work new questions to which no sufficient answers had yet been given seemed continually to arise. The pleasure and labour of attempting to answer these, however imperfectly, has delayed the book.

Although a large portion of this volume has already appeared in the latter half of the third edition, yet much of this has been recast and new illustrations and explanations have been given wherever they appeared to be necessary. Besides this much new matter has been added. Exactly also as in the last edition those parts to which the student should first turn his attention are printed in a larger type than the rest.

Following the same plan as in Vol. I., the several Chapters have been made as independent as possible. The object in view was that the reader should select his own order of study. Historical notices and references have been given throughout the book. But it has not been thought necessary to refer to the author's own additions to the subject, except when they have been first published elsewhere.

In this volume much use has been made of the new symbol for a fraction lately introduced by Prof. Stokes. The symbol

a/b for $\frac{a}{b}$ is very convenient as it enables the algebraical formulæ to be written on a line with the type. If some such abbreviation as this is not used two whole lines are required to write the simplest fraction. When the numerator or denominator of the fraction so written contains several factors, the rule adopted has been that all that follows the slant line up to the next plus or minus sign is to be regarded as the denominator. In the same way all that precedes the slant line up to the next plus or minus sign is to be taken as the numerator. When more complicated factors have to be written, brackets are used to indicate the numerator and denominator. Thus $\frac{ab}{cd}+\frac{e+f}{g-h}$ would be written

$$ab/cd+(e+f)/(g-h).$$

Numerous examples have been given throughout the book. Some of these are intended to be merely simple exercises, but many are important as illustrating and completing the theories given in the text. Sometimes when the principles of a theory had been explained numerous applications seemed to arise. Instead of loading the text with these it appeared preferable to put them into the form of examples and to give such hints as would make their solution easy. Everywhere the results have been given, and care has been taken to secure their accuracy; but amongst so many problems, it cannot be expected that no errors have escaped detection.

EDWARD J. ROUTH.

PETERHOUSE,
August, 1884.

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